|  | Chapter $\mathbf{1 1}$ Calendar |  |
| :---: | :---: | :---: |
| Day | Date | Assignment (Due the next class meeting) |
| Monday | $4 / 25 / 22$ | Exponential Review |
| Tuesday | $4 / 26 / 22$ | $\mathbf{1 1 . 1}$ Worksheet |
| Wednesday | $4 / 27 / 22$ | Introduction to Logarithmic Expressions |
| Thursday | $4 / 28 / 22$ | $\mathbf{1 1 . 2}$ Worksheet |
| Friday | $4 / 29 / 22$ | Graphing Logarithmic Expressions |
| Monday | $5 / 2 / 22$ | $\mathbf{1 1 . 3}$ Worksheet |
| Tuesday | $5 / 3 / 22$ | 11.4 Worksheet |
| Wednesday | $5 / 4 / 22$ | Properties of Logarithmic Expressions |
| Thursday | $5 / 5 / 22$ | Solving Logarithmic Equations |
| Friday | $5 / 6 / 22$ | 11.5 Worksheet |
| Monday | $5 / 9 / 22$ | Solving Exponential Equations |
| Tuesday | $5 / 10 / 22$ | Unit $\mathbf{1 1}$ Practice Test |
| Wednesday | $5 / 11 / 22$ | Unit 11 Review |
| Thursday | $5 / 12 / 22$ | Unit 11 Test |
| Friday | $5 / 13 / 22$ |  |
| Monday | $5 / 16 / 22$ |  |
| Tuesday | $5 / 17 / 22$ | $5 / 18 / 22$ |

* Be prepared for daily quizzes.
* Every student is expected to do every assignment for the entire unit.
* Try www.khanacademy.org if you need help outside of school hours.
* Student who complete $100 \%$ of their homework second semester will receive a pizza party and $2 \%$ bonus to their grade!


### 11.1 Notes: Introduction to Logarithmic Expressions

Can you find the inverse of $y=2^{x}$ ?

What can we say about Exponentials and Logarithms?

| Log Form: | Exponential Form: |
| :--- | :--- |
| Logarithm with Base b: |  |
| Logarithm with Base $\boldsymbol{e}$ : (Natural Log) $[l \mathrm{ln}]:$ |  |
|  |  |
| Logarithm with Base 10: (Common Log) |  |

Examples: Rewrite the following equations in logarithm form or exponential form.

1) $\log _{3} 81=4$
2) $\quad \log 1=0$
3) $\quad 2^{3}=8$
4) $(1 / 4)^{-1}=4$

You try! Rewrite the following equations in logarithm form or exponential form.
a) $\quad 4^{3}=64$
b) $\quad \log _{2} 32=5$
c) $\quad(1 / 2)^{-2}=4$

Examples: Evaluate the following without a calculator.
5) $\quad \log _{4} 64$
6) $\quad \log 0.001$
7) $\quad \log _{1 / 4} 256$
8) $\quad \log _{64} 2$

You try! Evaluate the following without a calculator.
a) $\log 100$
b) $\log _{27} 3$
c) $\log _{1 / 2} 8$
*Inverse functions: Logarithms and exponentials are inverses if they have the

| Properties of <br> Logs | $\log _{b} b^{m}=\quad$ | $\log _{b} 1=$ | $\log _{b} b=$ | $b^{\log _{b} m}=$ |
| :--- | :--- | :--- | :--- | :--- |

Examples: Simplify the following expressions.
9) $10^{\log 4}$
10) $\mathrm{e}^{\ln 9}$
11) $\log _{5} 5^{3}$
12) $\quad \log _{3} 27^{x}$
13) $\quad \log _{5} 25^{-2 x}$
14) $\quad \ln e^{-2 x}$
15) $\log _{2} 16+\log _{5} 625$
16) $\quad \log _{3} 27-\ln \mathrm{e}+\log _{2} 32$

You try! Simplify the following expressions.
a) $3^{\log _{3} 81}$
b) $e^{\ln 12}$
c) $\log _{4} 64^{3 x}+\ln e^{2 x}$
d) $\log _{2} 64-\log _{1 / 5} 25$

### 11.2 Notes: Graphs of Logarithmic Equations

a) Make a table and graph $y=2^{x}$.

What is the Domain?
What is the Range?

b) Now switch the input and output values (x's and y's) and graph them on the same grid. What are you graphing?
c) What is the domain and range for the function graphed in part b?
d) Fill in the table and graph $y=\log _{2} x$. What do you notice?

| $\mathbf{x}$ | $\mathbf{y}$ |
| :---: | :---: |
| $1 / 4$ |  |
| $1 / 2$ |  |
| 1 |  |
| 2 |  |
| 4 |  |

So, $y=2^{x}$ and $y=\log _{2} x$ are $\qquad$ .

Summarize the transformations for the graph of a logarithmic equation:

$$
f(x)=a \log _{b}(x-h)+k
$$

| Steps to Graph Logarithmic Functions: |
| :---: |
| $\boldsymbol{y}=\boldsymbol{a} \cdot \boldsymbol{l o g}_{\boldsymbol{b}}(\boldsymbol{x}-\boldsymbol{h})+\boldsymbol{k}$ |$|$| Hint: Identify all transformations first |
| :---: |
|  |
|  |
|  |

Examples: Graph the following functions and state the domain and range in set notation.

1) $\quad f(x)=\log _{2}(x+3)-1$


Domain:
Range:
3) $y=-\log _{2}(x-3)$


Domain:
Range:
5)


Domain:
Range:
2) $f(x)=\log _{3}(x+2)+4$


Domain:

## Range:

4) $f(x)=\log _{2} x-2$


Domain:
Range:
6) $\quad f(x)=2 \log _{3}(x+1)-5$


Domain:
Range:
7) $\quad f(x)=\frac{1}{3} \log _{2}(x+3)-1$


Domain:
Range:
8) $\quad f(x)=-3 \log _{2}(x-4)$


Domain:
Range:

### 11.3 Notes: Properties of Logarithmic Expressions

Product Property:

Quotient Property:

Power Property:

Examples: Use properties of logarithms to evaluate the following Use $\log _{6} 5 \approx 0.898$ and $\log _{6} 8 \approx 1.161$ to evaluate the logarithm

1) $\quad \log _{6}\left(\frac{5}{8}\right)$
2) $\quad \log _{6} 40$
3) $\quad \log _{6} 125$

You try! Use properties of logarithms to evaluate the following Use $\log _{3} 6 \approx 1.631$ and $\log _{3} 2 \approx 0.631$ to evaluate the logarithm
a) $\log _{3} 12$
b) $\log _{3} 8$
c) $\log _{3}\left(\frac{2}{6}\right)$

Examples: Condense each logarithmic expression.
4) $2 \log x+3 \log y-2 \log z$
5) $\ln 4+3 \ln 3-\ln 12+6$

You try! Condense each logarithmic expression.
a) $\quad \log _{5} 3-4 \log _{5} a+5 \log _{5} b$
b) $3 \ln 2-\ln 6-\ln 4$

Examples: Expand each logarithmic expression.
6) $\log _{7} \frac{3 x}{5 y}$
7) $\ln \frac{2 x^{2}}{5 y}$

You try! Expand each logarithmic expression.
a) $\log \frac{2 a b^{5}}{3 c^{3}}$
b) $\ln \frac{5 x}{y z^{4}}$

## Change-of-base Formula

Examples: Use the change-of-base formula to evaluate each logarithm. Give an exact answer and an approximate solution to three decimal places.
8) $\log _{5} 8$
9) $\quad \log _{8} 14$
10) $\quad \log _{3} 12$

You try! Use the change-of-base formula to evaluate each logarithm. Give an exact answer and an approximate solution to three decimal places.
a) $\log _{4} 15$
b) $\log _{3} 2$
c) $\log _{100} 1$

### 11.4 Notes: Solving Logarithmic Equations

Property of Equality for Logarithmic Equations

Examples: Solve each logarithmic equation and check for extraneous solutions.

1) $\quad \log _{5}(4 \mathrm{x}-7)=\log _{5}(\mathrm{x}+5)$
2) $\quad \log _{4}(5 x-1)=3$
3) $\quad \ln (7 x-4)=\ln (2 x+11)$
4) $\log _{3}(x-1)+\log _{3} x=\log _{3} 20$
5) 

$\log _{4}(\mathrm{x}+12)+\log 4 \mathrm{x}=3$
6) $10=\log _{2}(x-3)+6$

You try! Solve each logarithmic equation and check for extraneous solutions.
a) $\log (5 x-21)=\log (3 x-4)$
b) $\ln (x-2)+\ln x=\ln 8$
c) $3 \log _{2}(x-7)=24$
d) $\log x+\log (x+15)=2$
8) The population of deer in a forest preserve can be modeled by the equation $P=50+200 \ln (t+1)$, where $t$ is the time in years from the present. In how many years will be deer population reach 500 ?
9) One of the strongest earthquakes in recent history occurred in Mexico City in 1985 and measured 8.1 on the Richter scale. Find the amount of energy, E, released by this earthquake with a magnitude of $M$.
Use the formula: $\quad M=\frac{2}{3} \log \frac{E}{10^{11.8}}$

### 11.5 Notes: Solving Exponential Equations

Examples: Solve the following equations by taking the logarithm of both sides or by re-writing it in logarithm form. Give an exact solution and an approximate solution rounded to three decimal places.

1) $4^{x}=11$
2) $7^{9 x}=15$
3) $4 e^{-0.3 \mathrm{x}}-7=13$

You try! Solve the following equations by taking the logarithm of both sides or by re-writing it in logarithm form. Give an exact solution and an approximate solution rounded to three decimal places.
a) $3 e^{2 x}+5=20$
b) $3^{2 x}=18$

## Work with a partner to answer this question. Be prepared to explain your process to the class.

You want to buy a car that is going to cost $\$ 4,000$. Right now you have saved $\$ 3,000$ but you want to invest your money to help is grow a little faster. The bank will offer you a savings account that earns $5 \%$ interest compounded annually. How long will it take for you to have enough money for your car?

Example 4: If $\$ 5,000$ is deposited in an account at the bank and earns $7.5 \%$ annual interest, compounded annually, how long does it need to stay in the account in order to double? Use the formula $A=P\left(1+\frac{r}{n}\right)^{n t}$

Example 5: If $\$ 2,500$ is invested at a rate of $3 \%$ compounded continuously, find the amount of time for the account to have $\$ 3,300$ in it. Use the formula $A=P e^{r t}$

Example 6: Your freshman year of high school you were given some money to invest in a savings account. You wanted to have $\$ 3,000$ when you graduated in four years. The bank gave you an interest rate of $8 \%$ compounded continuously. How much money did you invest? Use the formula $A=P e^{r t}$

## You try!

a) Samantha and Ryan are having a contest to see who can double their investment first. Both have $\$ 250$ to deposit. Samantha puts her money into an account with an $8.5 \%$ interest rate, compounded continuously. Ryan puts his money into an account with a $9 \%$ interest rate, compounded quarterly. Who will double their investment first? Explain your answer.

## Example 7:

Write an exponential function in the form $y=a b^{x}$ whose graph passes through the points $(2,12.5)$ and $(4,312.5)$.
A. $y=\frac{1}{5}(2)^{x}$
B. $y=\frac{1}{2}(5)^{x}$
C. $y=2(5)^{x}$
D. $y=5(2)^{x}$

